

# **Electrical Theory**

## **Mathematics Review**

**PJM State & Member Training Dept.**

# Objectives



By the end of this presentation the Learner should be able to:

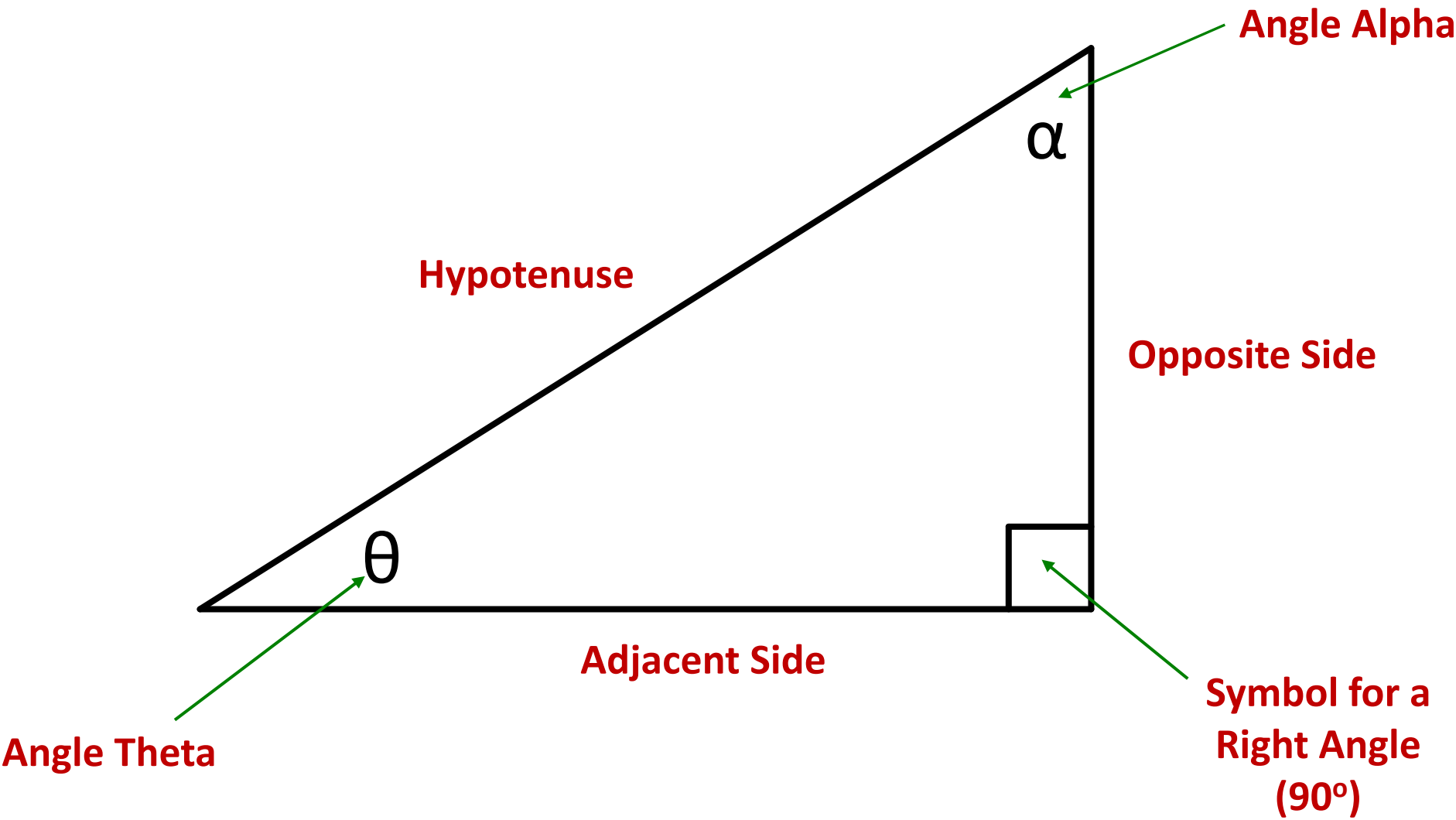
- Use the basics of trigonometry to calculate the different components of a right triangle
- Compute Per-Unit Quantities
- Identify the two components of Vectors

# Right Triangles

# Mathematics Review

- To be able to understand basic AC power concepts, a familiarization with the relationships between the angles and sides of a right triangle is essential
- A right triangle is defined as a triangle in which one of the three angles is equal to  $90^\circ$

# Mathematics Review



# Mathematics Review

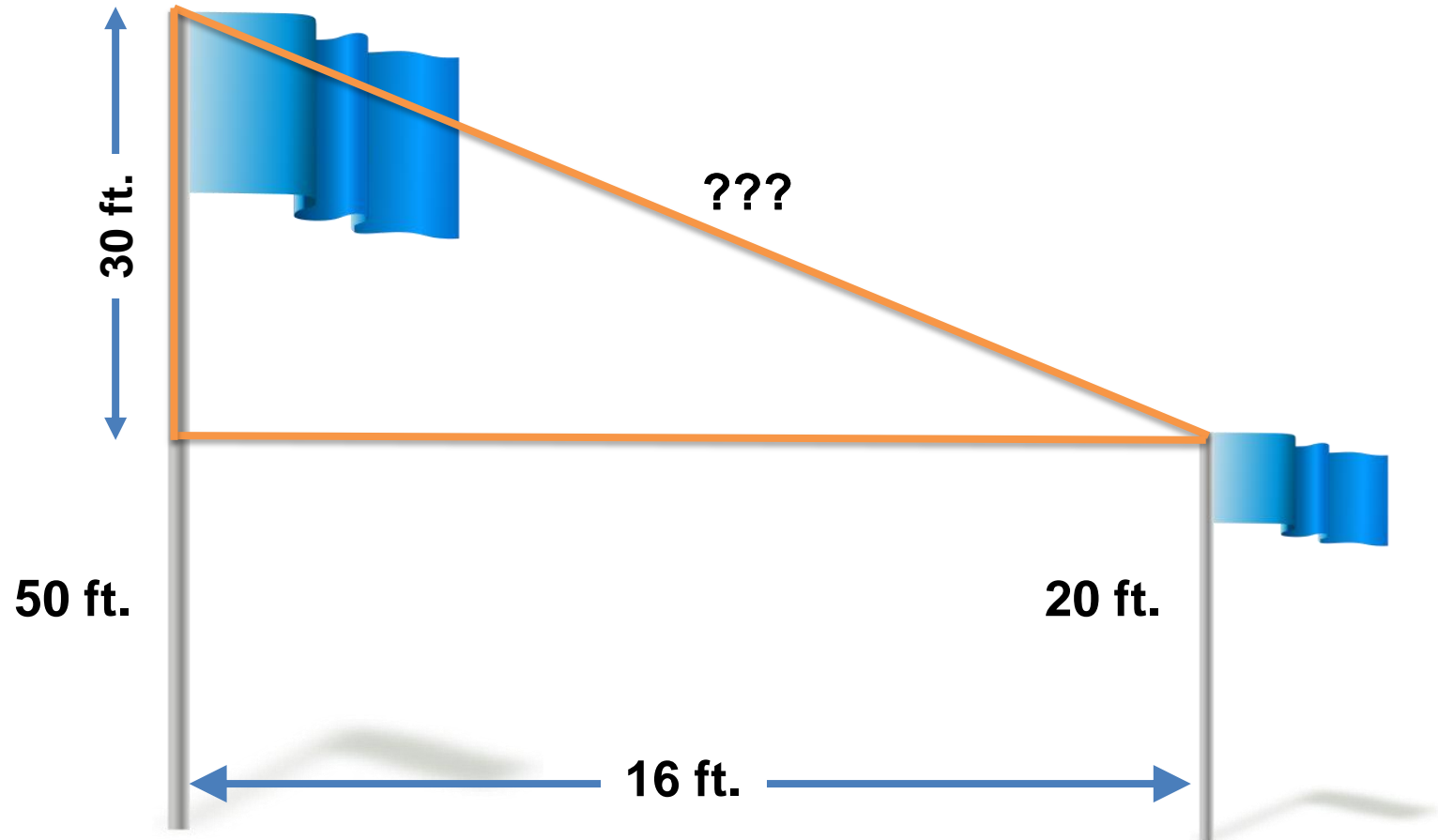
- Given the lengths of two sides of a right triangle, the third side can be determined using the **Pythagorean Theorem**

$$\text{Hypotenuse}^2 = \text{Opposite}^2 + \text{Adjacent}^2$$

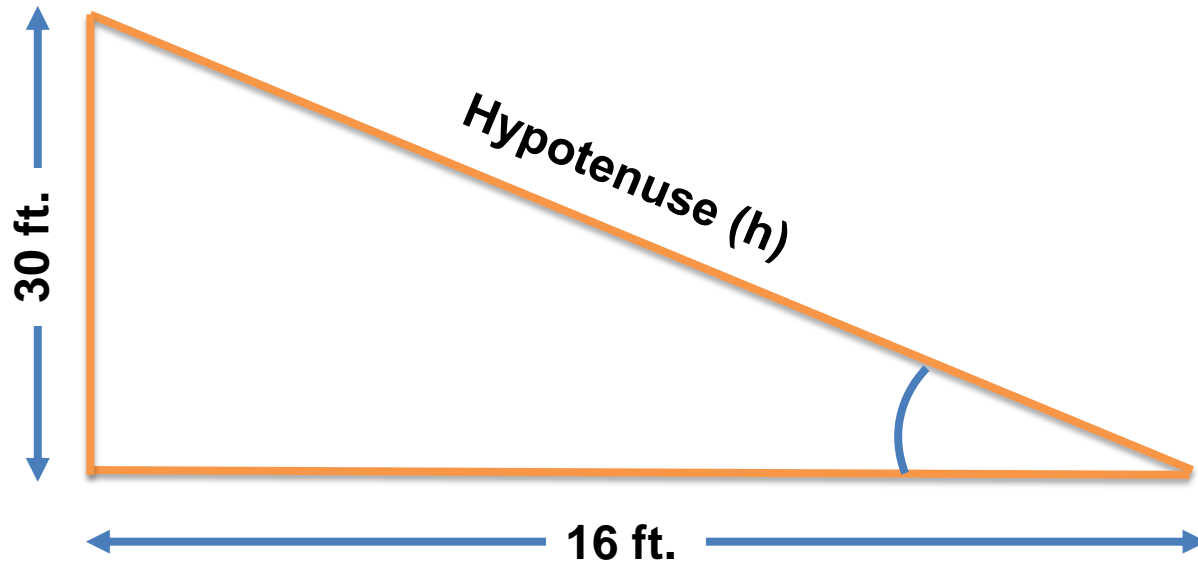
## Example:

A rope stretches from the top of one pole 50 feet high to the top of another pole 20 feet high, standing 16 feet away.

*How long is the rope?*



## Example:



$$\begin{aligned}h^2 &= a^2 + o^2 \\h^2 &= 16^2 + 30^2 \\h^2 &= 256 + 900 \\h^2 &= 1156 \\h &= \sqrt{1156} \\h &= 34\end{aligned}$$



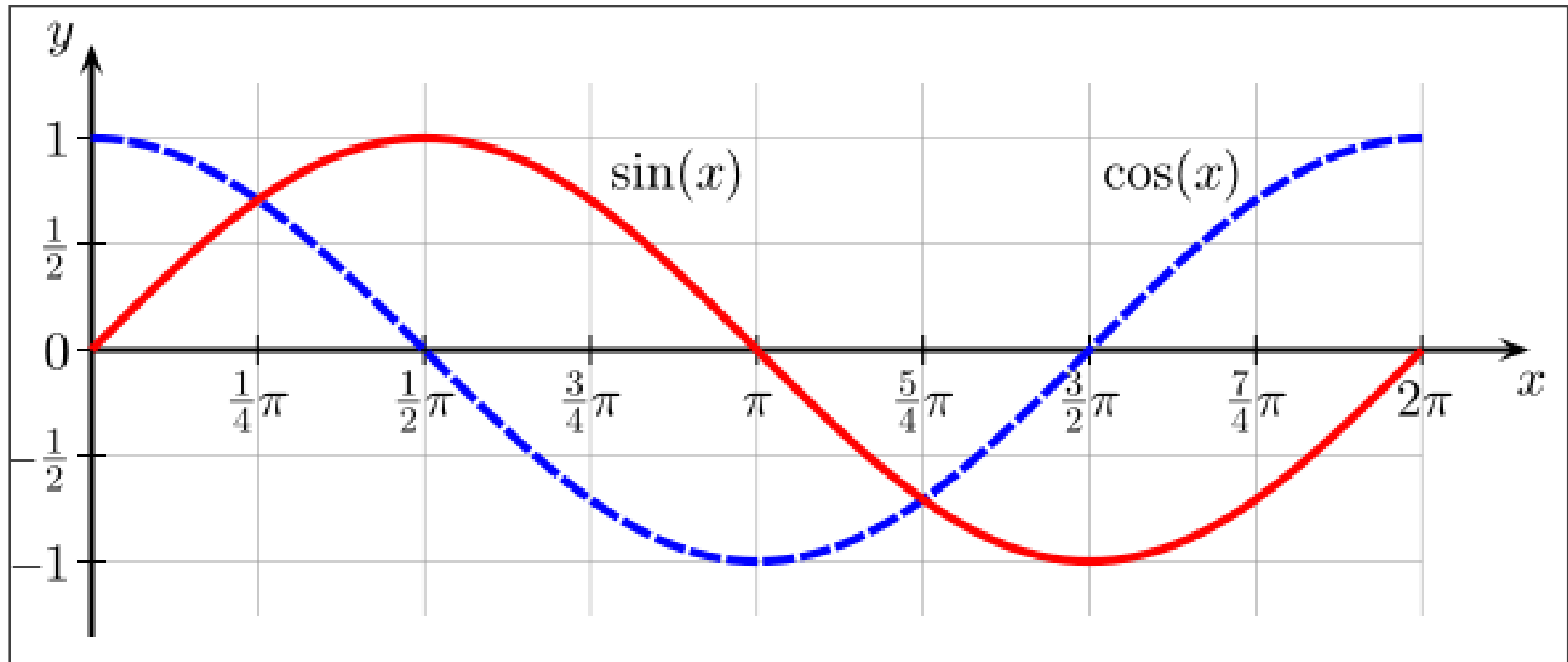
# Mathematics Review

- Once the sides are known, the next step in solving the right triangle is to determine the two unknown angles of the right triangle
- All of the angles of any triangle always add up to  $180^\circ$
- In solving a right triangle, the remaining two unknown angles must add up to  $90^\circ$
- Basic trigonometric functions are needed to solve for the values of the unknown angles

# Trigonometry

# Mathematics Review

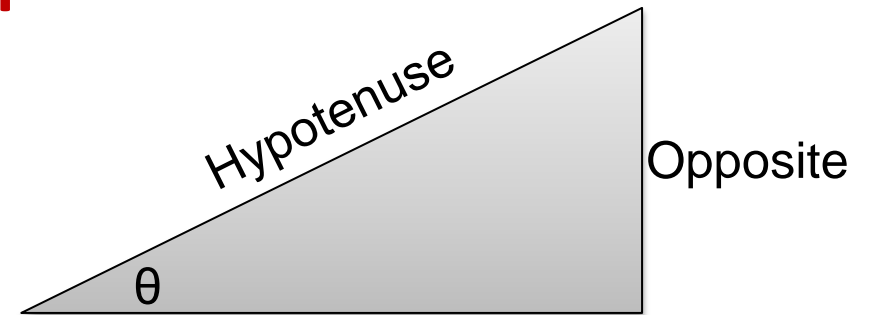
- The sine function is a periodic function in that it continually repeats itself



# Mathematics Review

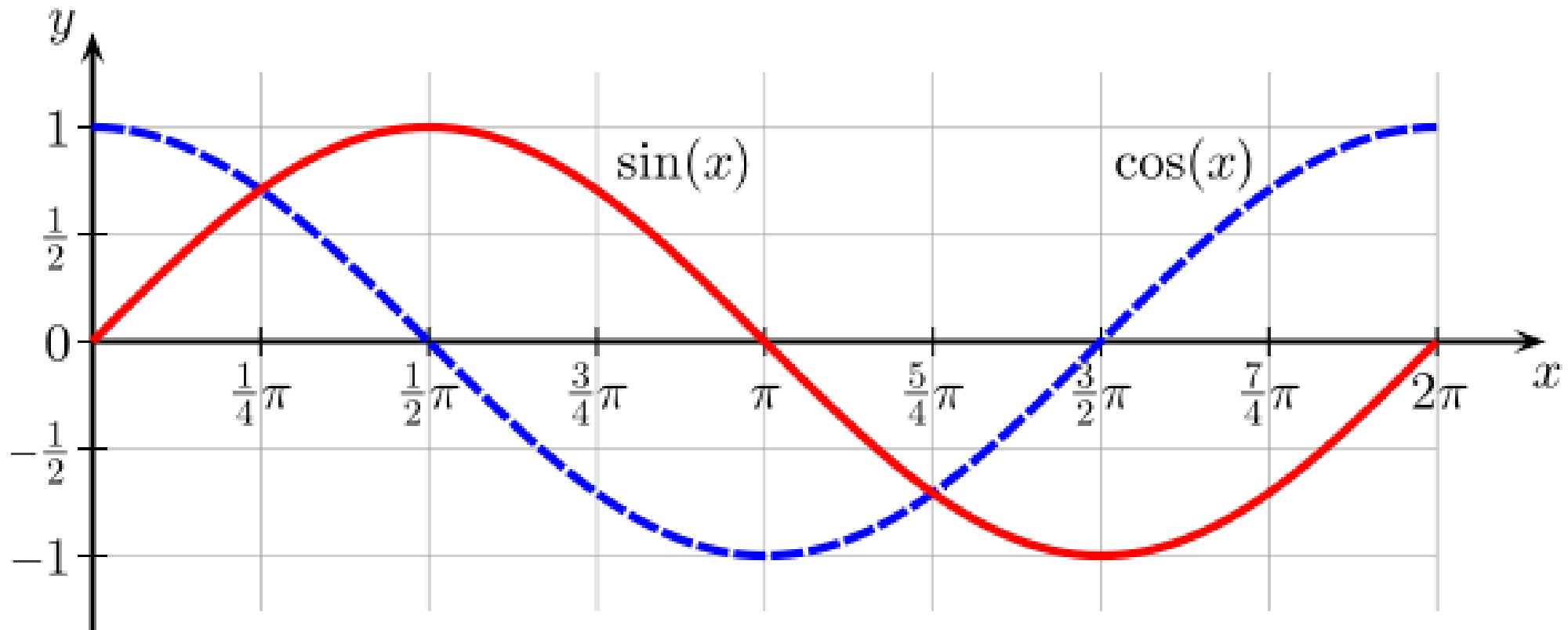
- In order to solve right triangles, it is necessary to know the value of the sine function between  $0^\circ$  and  $90^\circ$
- Sine of either of the unknown angles of a right triangle is the ratio of the length of the opposite side to the length of the hypotenuse

$$\text{SIN } \theta = \text{Opposite Side} / \text{Hypotenuse}$$



# Mathematics Review

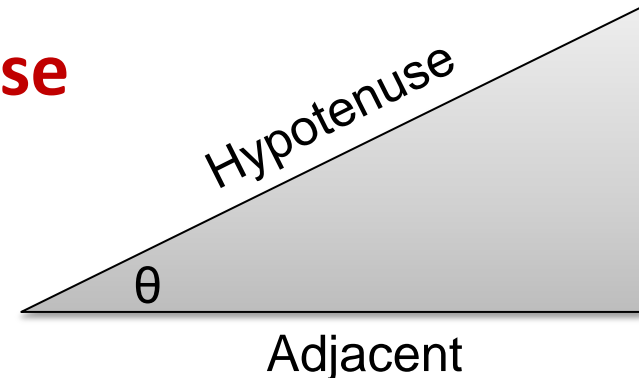
- Cosine function is a periodic function that is identical to the sine function except that it leads the sine function by  $90^\circ$



# Mathematics Review

- As an example, the cosine function at  $0^\circ$  is 1 whereas the sine function does not reach the value of 1 until  $90^\circ$
- Cosine function of either of the unknown angles of a right triangle is the ratio of the length of the adjacent side to the length of the hypotenuse

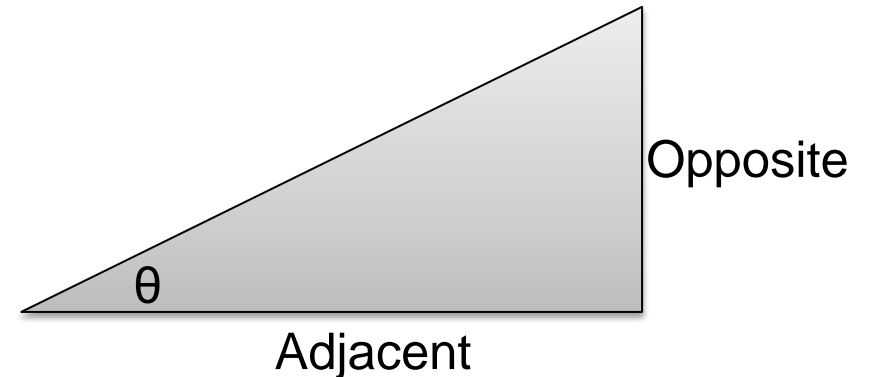
$$\cos \theta = \text{Adjacent Side} / \text{Hypotenuse}$$



# Mathematics Review

- The tangent function of either of the unknown angles of a right triangle is the ratio of the length of the opposite side to the length of the adjacent side

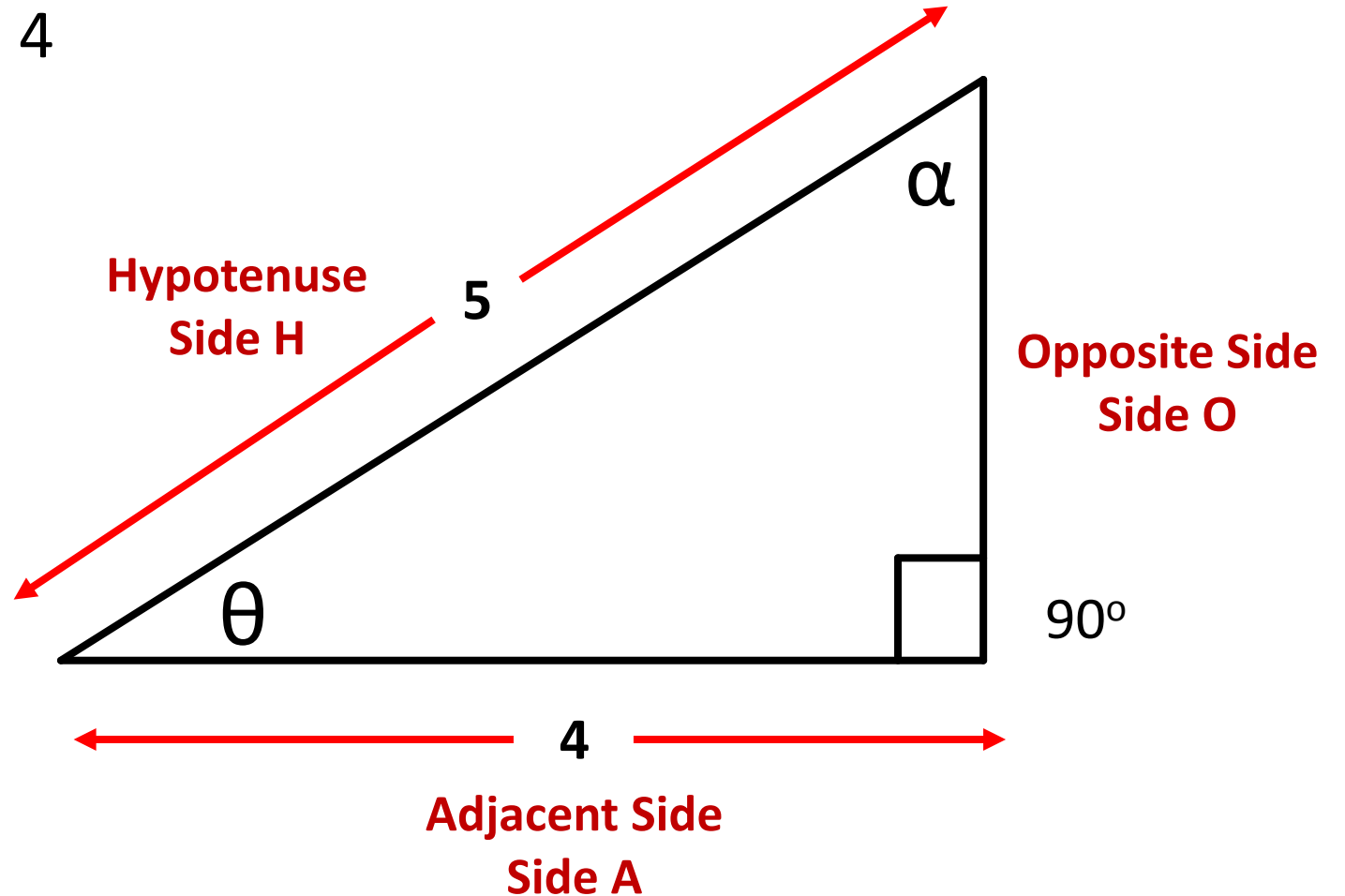
$$\text{TAN } \theta = \text{Opposite Side} / \text{Adjacent Side}$$



# Mathematics Review

Example:

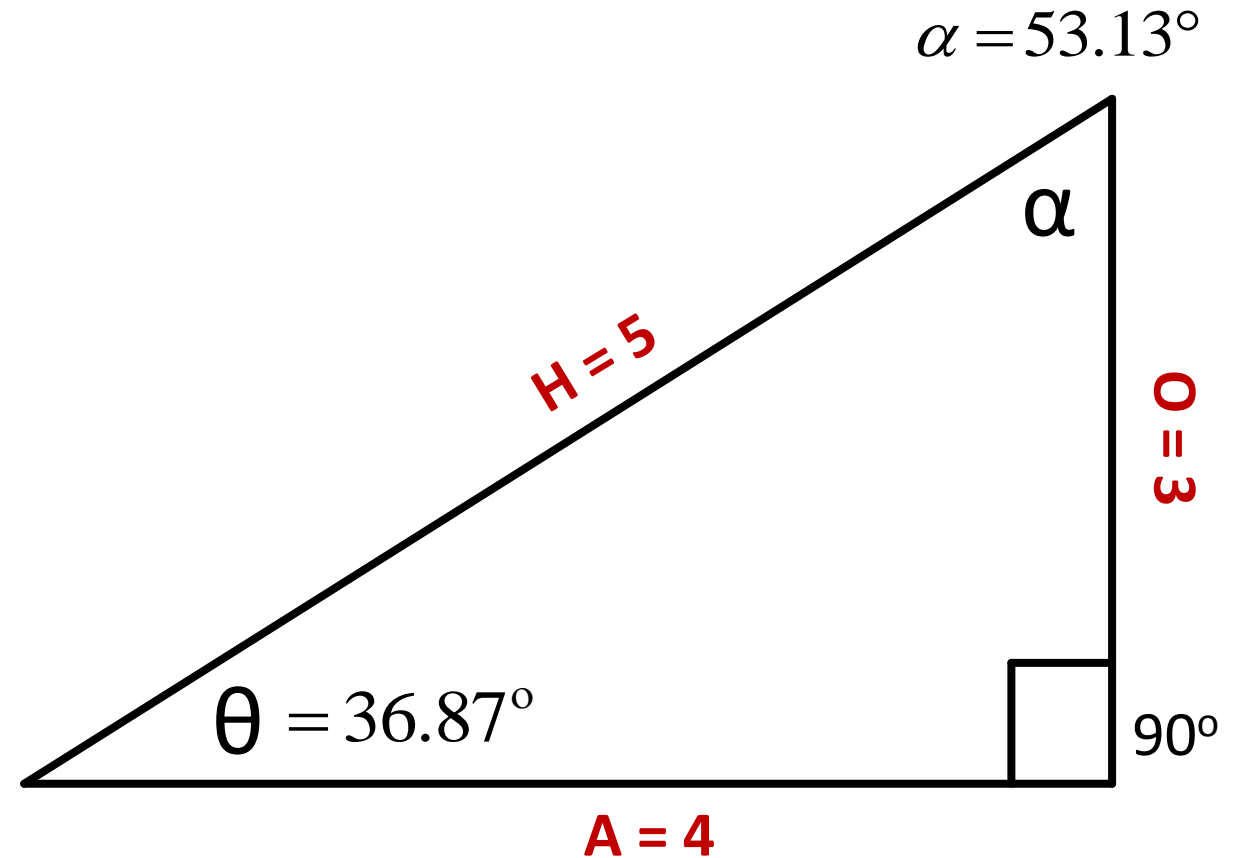
- Given: Side H = 5, Side A = 4
- Find: Side O, Angle  $\theta$  and Angle  $\alpha$





# Mathematics Review

- Find Side O:  $H^2 = A^2 + O^2$   
 $25 = 16 + O^2$   
 $25 - 16 = O^2$   
 $\sqrt{25 - 16} = O$   
 $O = \sqrt{9} = 3$
- Find  $\theta$ :  $\cos \theta = \frac{A}{H} = \frac{4}{5} = .8$   
 $\cos^{-1}(.8) = 36.87^\circ$
- Find  $\alpha$ :  $180^\circ - 90^\circ - 36.87^\circ = 53.13^\circ$



# Per-Unit Quantities

# Mathematics Review

- Ratios play an important part in estimating power system performance
  - Relationship between two quantities as a fraction
  - Used when the relationship of two pairs of values is the same, and one of two similarly related values is known
    - Example: if you can drive 120 miles in 2 hours, how many miles could you drive in 8 hours?

$$\frac{120 \text{ miles}}{2 \text{ hours}} = \frac{X \text{ miles}}{8 \text{ hours}} = \frac{120 \text{ miles}(\cancel{8 \text{ hours}})}{2 \cancel{\text{hours}}} = \frac{X \text{ miles} (\cancel{8 \text{ hours}})}{\cancel{8 \text{ hours}}} = 480 \text{ miles}$$

# Mathematics Review

- Quantities on the power system are often specified as a percentage or a **per-unit** of their base or nominal value
  - Makes it easier to see where a system value is in respect to its base value
  - How it compares between different parts of the system with different base values
  - Allow for a dispatcher to view the system and quickly obtain a feel for the voltage profile

# Mathematics Review

- Assume that, at a certain substation, the voltage being measured is 510 kV on the 500 kV system. What is its per-unit value with respect to the nominal voltage?

Base or nominal voltage = 500 kV

Measured voltage = 510 kV

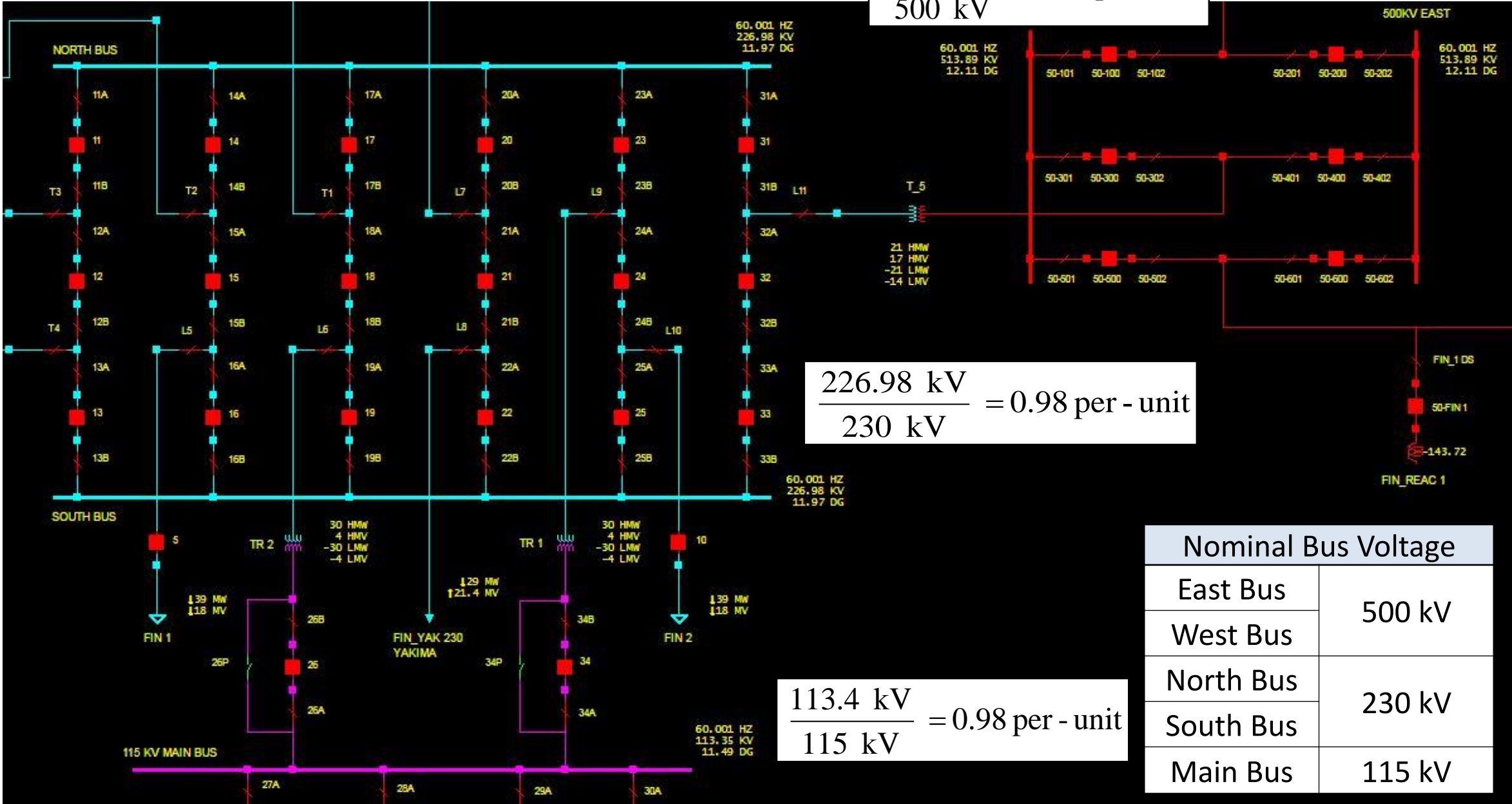
$510 \text{ kV} / 500 \text{ kV} = \underline{1.02 \text{ per-unit}}$  or 102%

# Mathematics Review

$$\frac{513.9 \text{ kV}}{500 \text{ kV}} = 1.03 \text{ per - unit}$$

$$\frac{226.98 \text{ kV}}{230 \text{ kV}} = 0.98 \text{ per - unit}$$

$$\frac{113.4 \text{ kV}}{115 \text{ kV}} = 0.98 \text{ per - unit}$$



## Question 4

- Assume that the loss of a 1000 MW generating unit will typically result in a 0.2 Hz dip in system frequency
- Estimate the frequency dip for the loss of an 800 MW generating unit

# Mathematics Review

$$\frac{0.2Hz}{1000MW} = \frac{(x)Hz}{800MW}$$



## Question 5

- Assume that the loss of a 500 MW generating unit will typically result in a 0.3 Hz dip in system frequency.
- Estimate the frequency dip for the loss of an 300 MW generating unit.

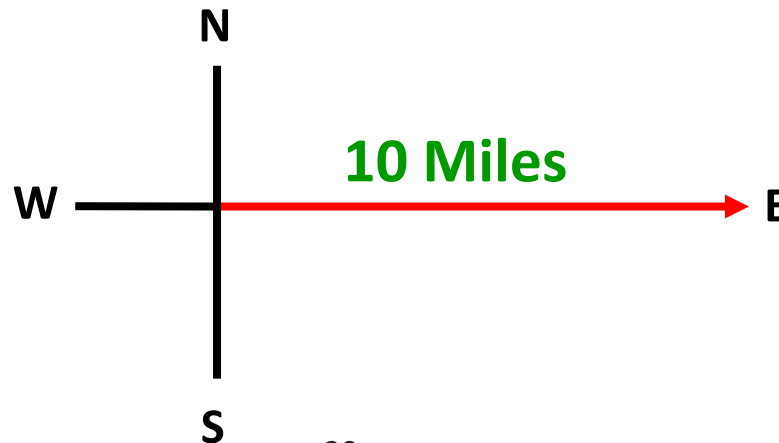
# Mathematics Review

$$\frac{0.3Hz}{500MW} = \frac{(x)Hz}{300MW}$$

# Vectors

# Vectors

- A vector is alternative way to represent a sinusoidal function with amplitude, and phase information
- Vectors are usually written in bold with an arrow over the top of the letter, ( $\vec{\mathbf{A}}$ )
- A vectors length represents **magnitude**
- A vectors **direction** represents the phase angle
- Example: 10 miles east



# Vectors

- Horizontal line to the right is positive; horizontal line to the left is negative
- Vertical line going up is positive; vertical line going down is negative
- Arrowhead on the end away from the point of origin indicates the direction of the vector and is called the displacement vector
- Vectors can go in any direction in space

# Vectors

- The difference between a scalar quantity and a vector:
  - a) A scalar quantity is one that can be described with a single number, including any units, giving its size or magnitude
  - b) A vector quantity is one that deals inherently with both magnitude and direction

## Conceptual Question 6

- There are places where the temperature is  $+20^{\circ}\text{C}$  at one time of the year and  $-20^{\circ}\text{C}$  at another time.
- Do the plus and minus signs that signify positive and negative temperatures imply that temperature is a vector quantity?

## Question 7

Which of the following statements, if any, involves a vector?

- a) I walked two miles along the beach
- b) I walked two miles due north along the beach
- c) A ball fell off a cliff and hit the water traveling at 17 miles per hour
- d) A ball fell off a cliff and traveled straight down 200 feet
- e) My bank account shows a negative balance of -25 dollars



# Vectors

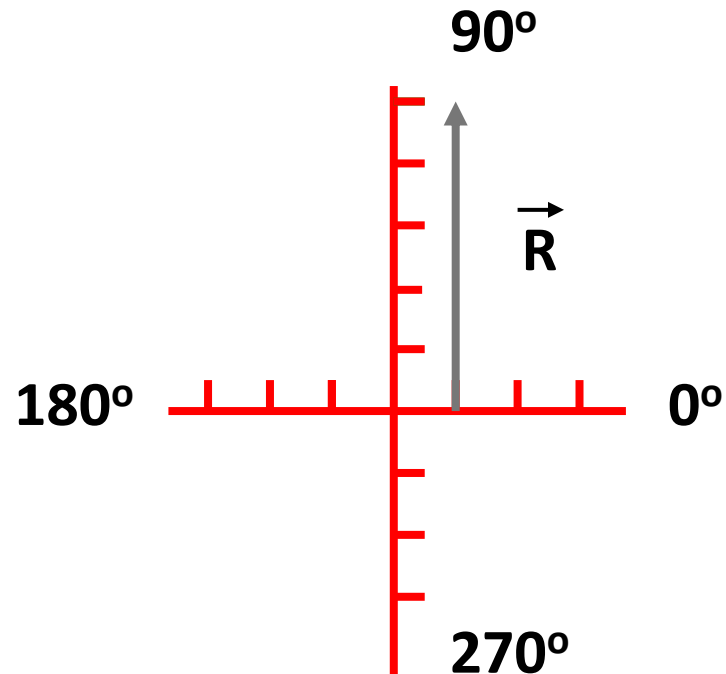
- When adding vectors, the process must take into account both the magnitude and direction of the vectors
- When adding two vectors, there is always a resultant vector,  $R$ , and the addition is written as follows:

$$\vec{R} = \vec{A} + \vec{B}$$

# Vectors

Example: Adding vectors in the same direction

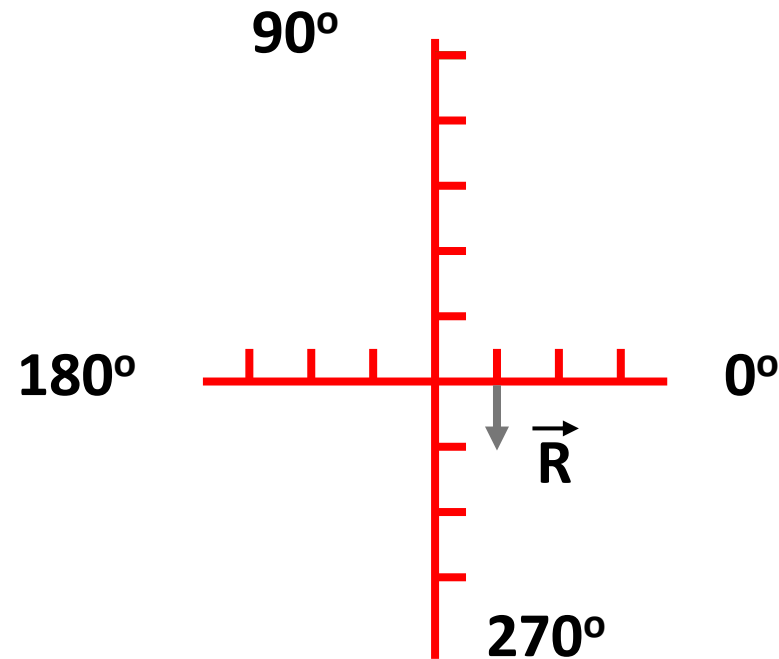
- Vector A has a length of 2 and a direction of  $90^\circ$
- Vector B has a length of 3 and a direction of  $90^\circ$



# Vectors

Example: Adding vectors in the opposite direction

- Vector  $\vec{A}$  has a length of 2 and a direction of  $90^\circ$
- Vector  $\vec{B}$  has a length of 3 and a direction of  $270^\circ$



# Answer Question 8

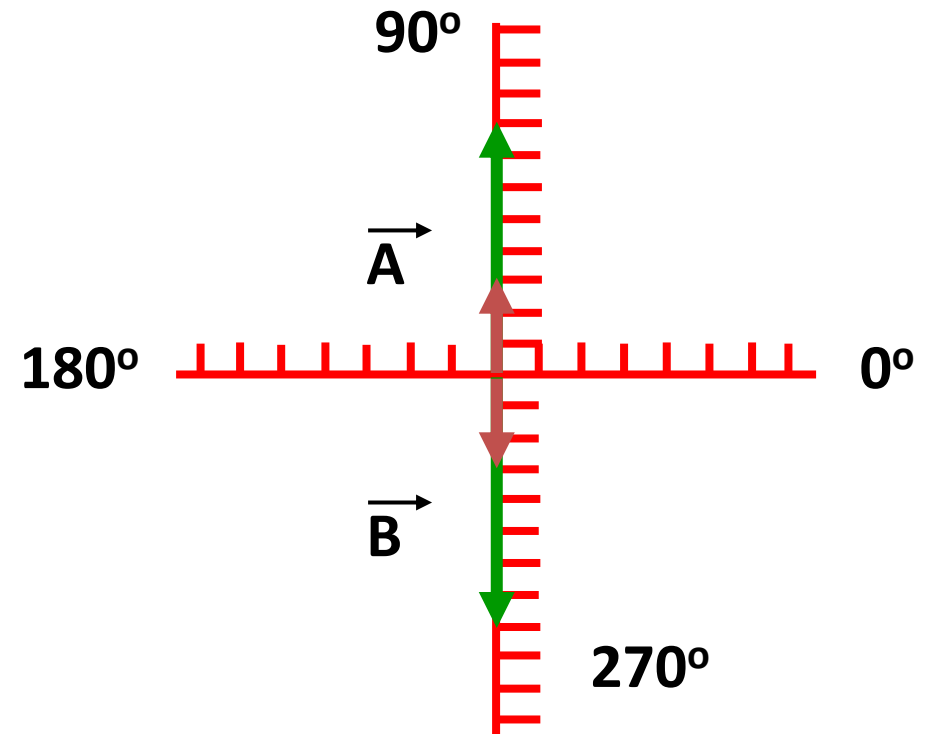
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## Question 8

Two vectors,  $\vec{A}$  and  $\vec{B}$ , are added to give a resultant vector,  $\vec{R}$ . The magnitudes are 3 and 8 meters, respectively, but the vectors can have any orientation. What is the maximum and minimum possible values for the magnitude of  $\vec{R}$ ?

**Maximum:  $3+8=11$**

**Minimum:  $(-3)+(-8)=(-11)$**



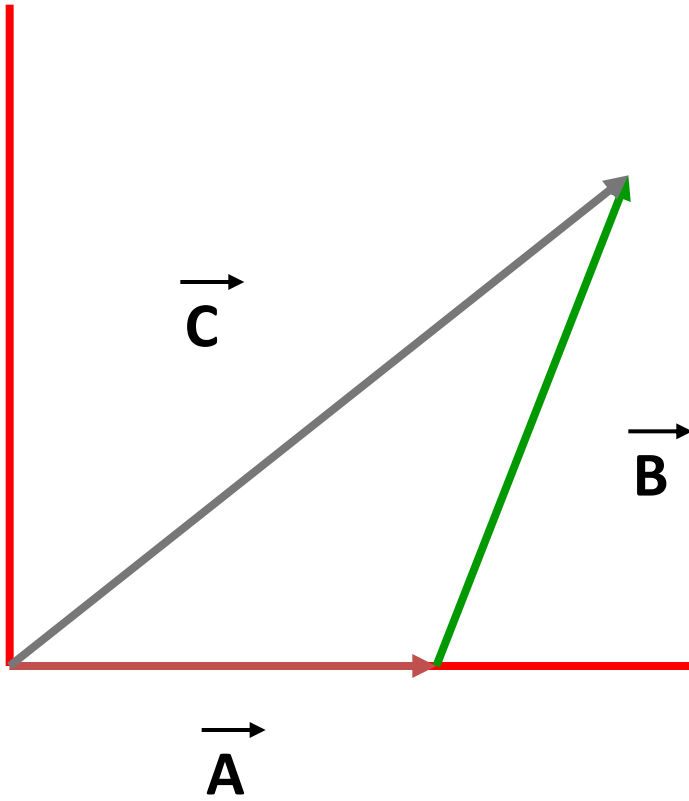
# Vectors

- Subtraction of one vector from another is carried out in a way that depends on the following:

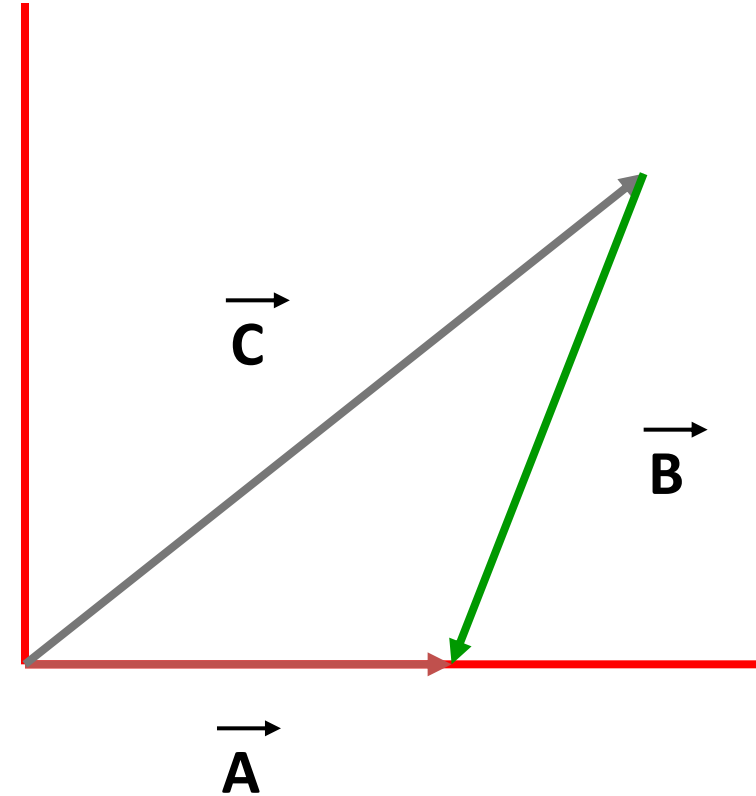
When a vector is multiplied by  $-1$ , the magnitude of the vector remains the same, but the direction of the vector is reversed

- Vector subtraction is carried out exactly like vector addition except that one of the vectors added is multiplied by the scalar factor of  $-1$

# Vectors



$$\vec{C} = \vec{A} + \vec{B}$$



$$\vec{A} = \vec{C} + (-\vec{B})$$

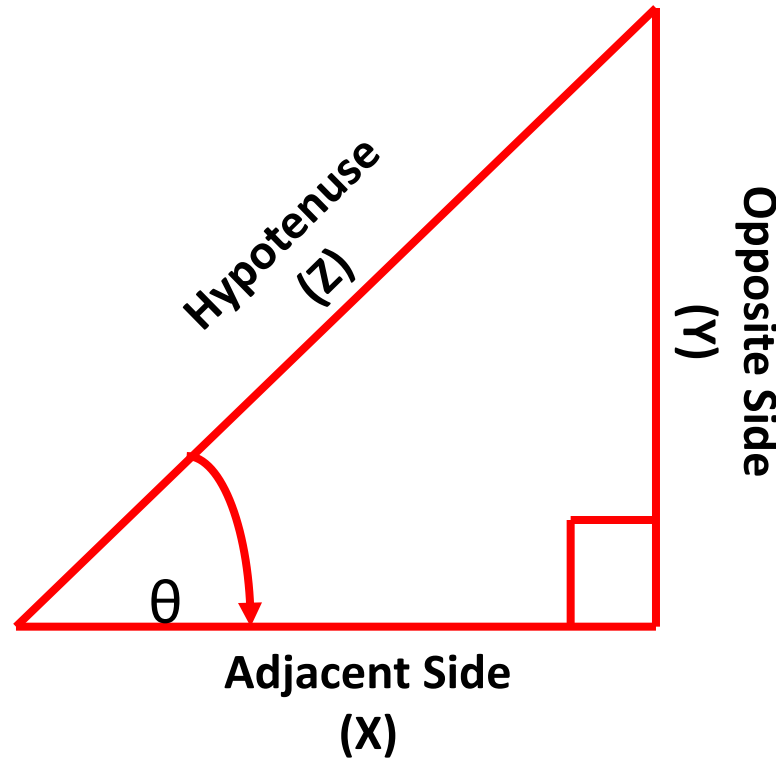
# Vectors

- If the magnitude and direction of a vector is known, it is possible to find the components of the vector
- The process is called “resolving the vector into its components”
- If the vector components are perpendicular and form a right triangle, the process can be carried out with the aid of trigonometry



# Vectors

- To calculate the sum of two or more vectors using their components (x and y) in the vertical and horizontal directions, trigonometry is used



# Vectors

- The Pythagorean Theorem is a special relationship that exists in any triangle and describes the relationship between the lengths of the sides of a right triangle

$$z^2 = x^2 + y^2$$

- Three basic trigonometric functions defined by a right triangle are:  
 $\sin \theta = y/z = \text{opposite side/hypotenuse}$   
 $\cos \theta = x/z = \text{adjacent side/hypotenuse}$   
 $\tan \theta = y/x = \text{opposite side/adjacent side}$   
 $\tan \theta = \sin \theta / \cos \theta$

# Vectors

- To find theta, the inverse of the trigonometric function must be used

$$\theta = \sin^{-1} y/z$$

$$\theta = \cos^{-1} x/z$$

$$\theta = \tan^{-1} y/x$$

- When adding vectors, magnitude is found by:

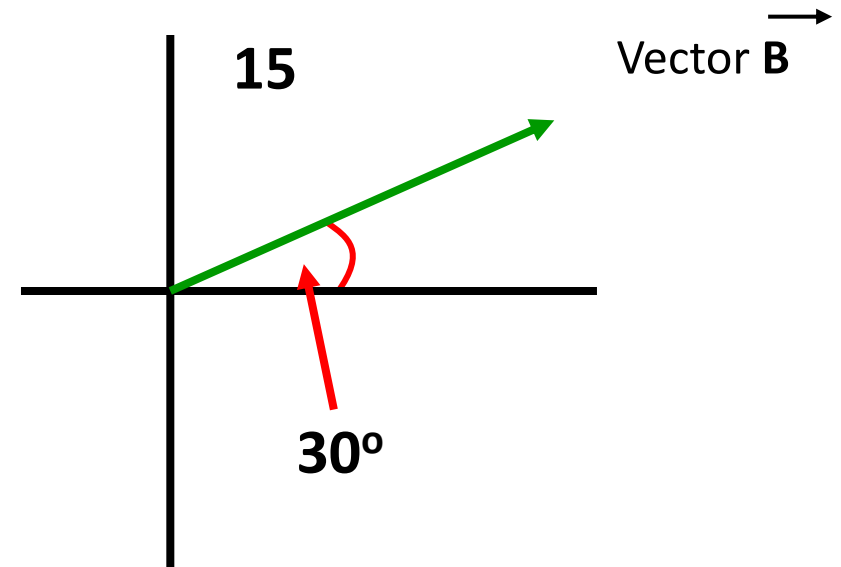
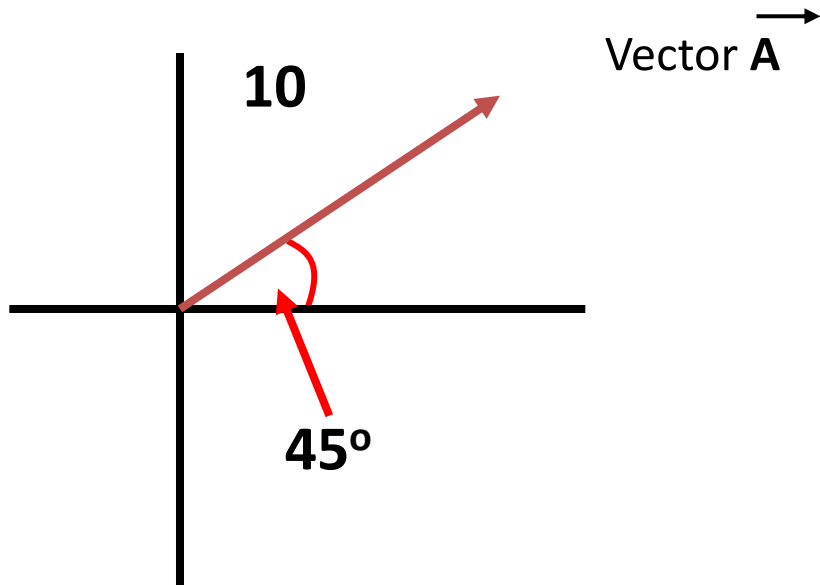
$$\vec{R} = \sqrt{(\vec{R}_x)^2 + (\vec{R}_y)^2}$$

- The direction of the resultant, R, is found by:

$$\theta = \sin^{-1} (R_y/R)$$

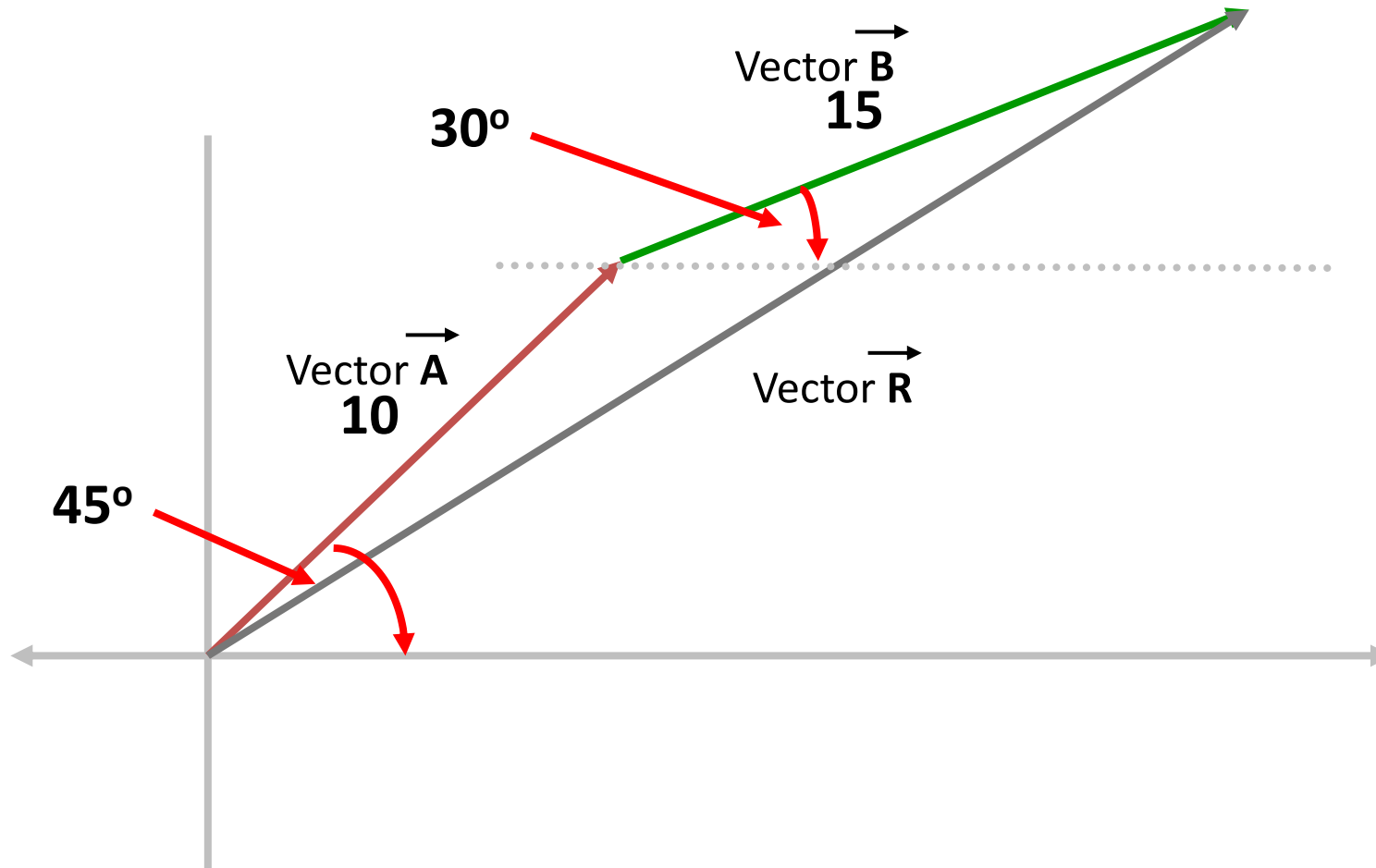
# Vectors

- Vectors can be added either in the same direction or in opposite directions

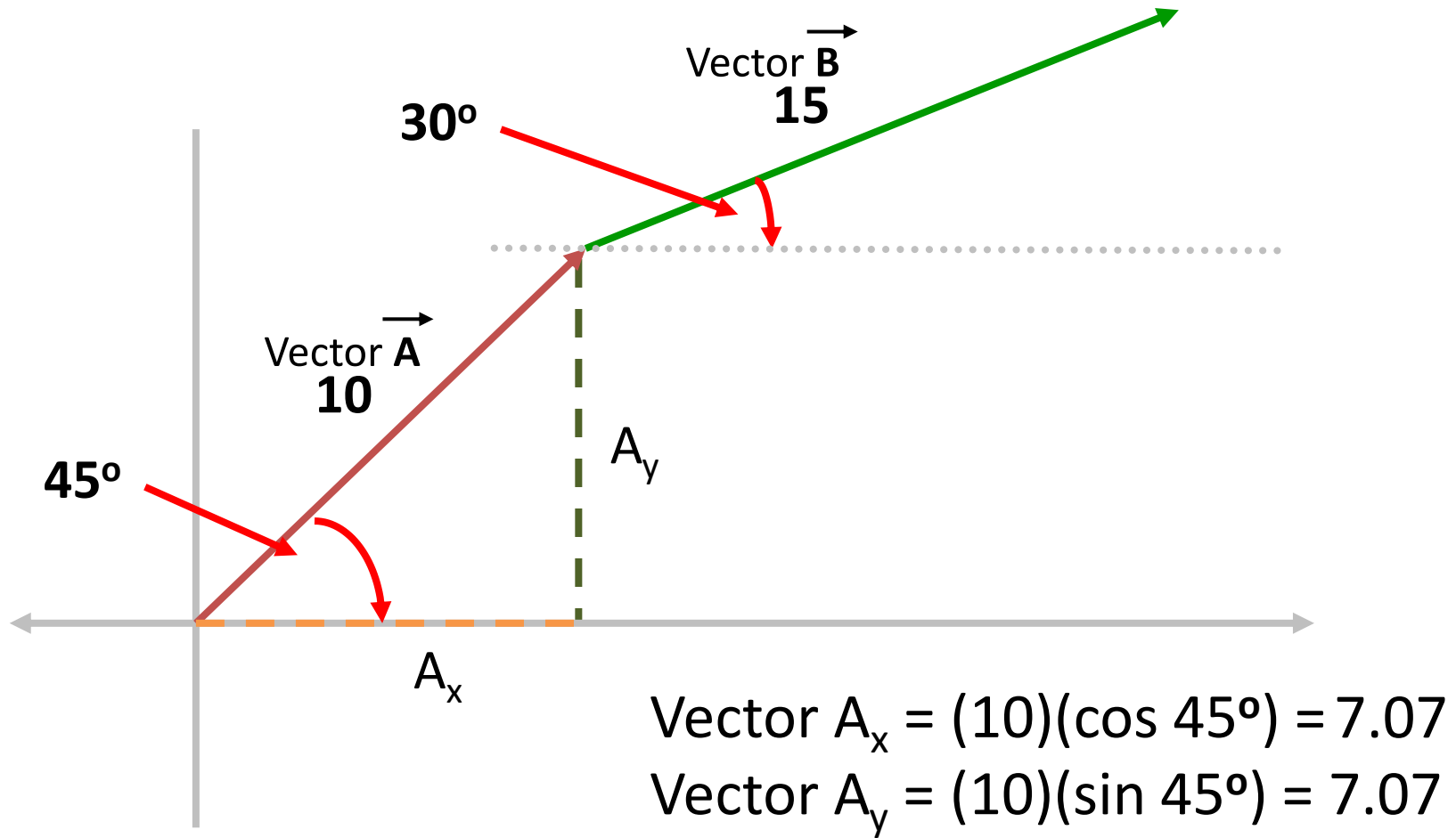


# Vectors

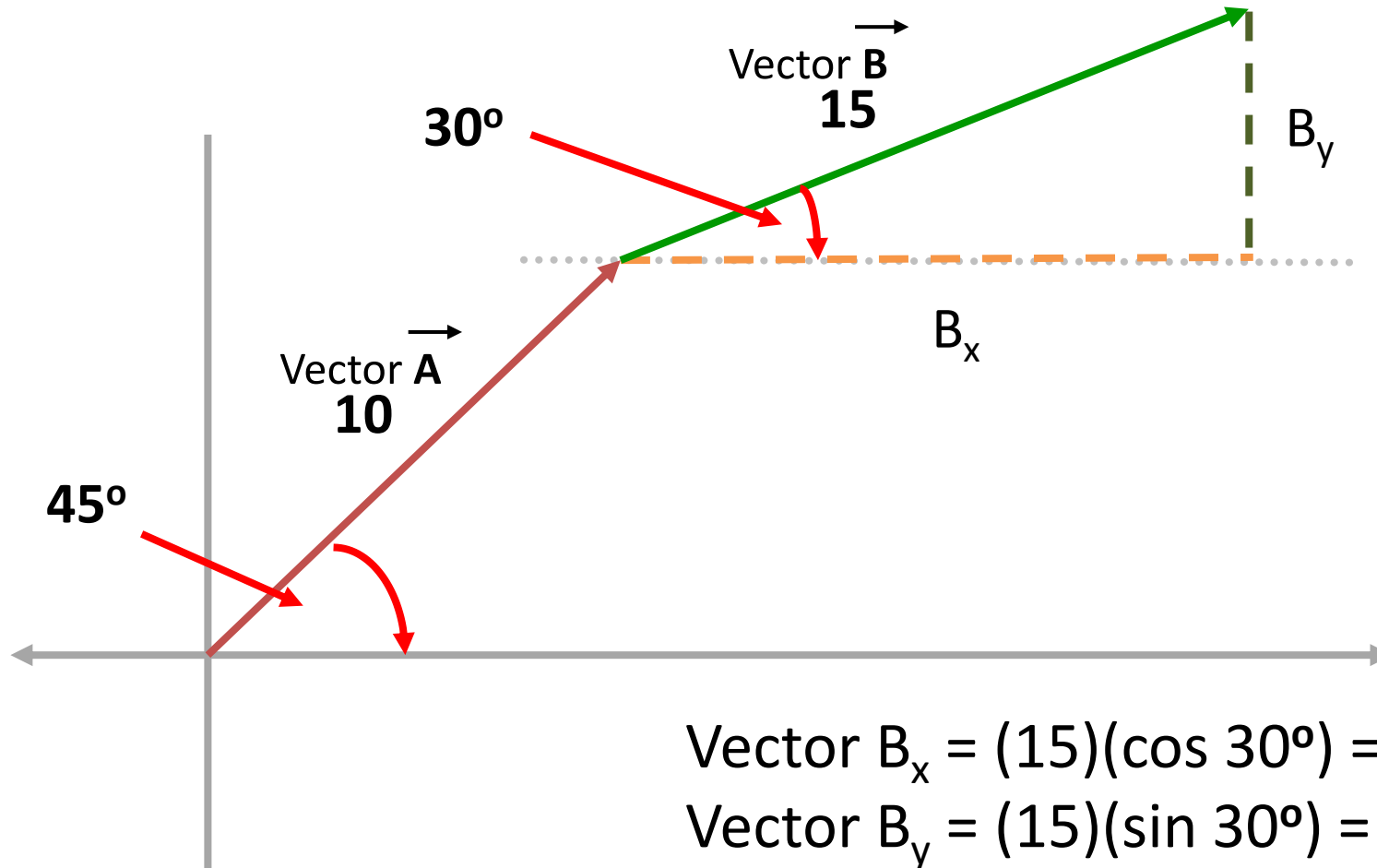
Adding vectors:



# Vectors



# Vectors



# Vectors

Determine the resulting vectors,  $R_x$  and  $R_y$ :

$$\vec{R}_x = \vec{A}_x + \vec{B}_x$$

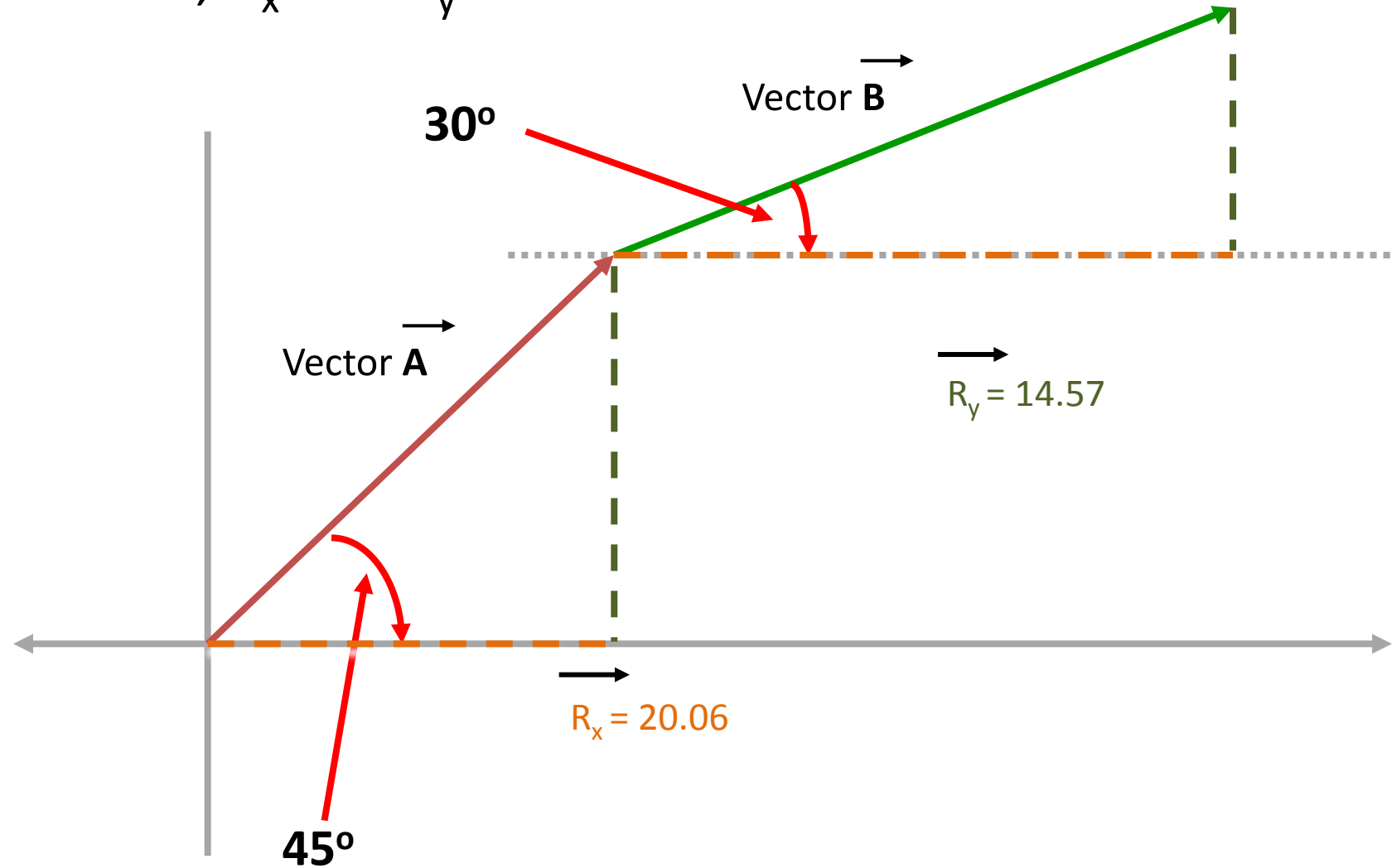
$$\vec{R}_x = 7.07 + 12.99$$

$$\vec{R}_x = 20.06$$

$$\vec{R}_y = \vec{A}_y + \vec{B}_y$$

$$\vec{R}_y = 7.07 + 7.5$$

$$\vec{R}_y = 14.57$$





# Vectors

$$\vec{R} = \sqrt{\vec{R}_x^2 + \vec{R}_y^2}$$

$$\vec{R} = \sqrt{20.06^2 + 14.57^2}$$

$$\vec{R} = \sqrt{614.7}$$

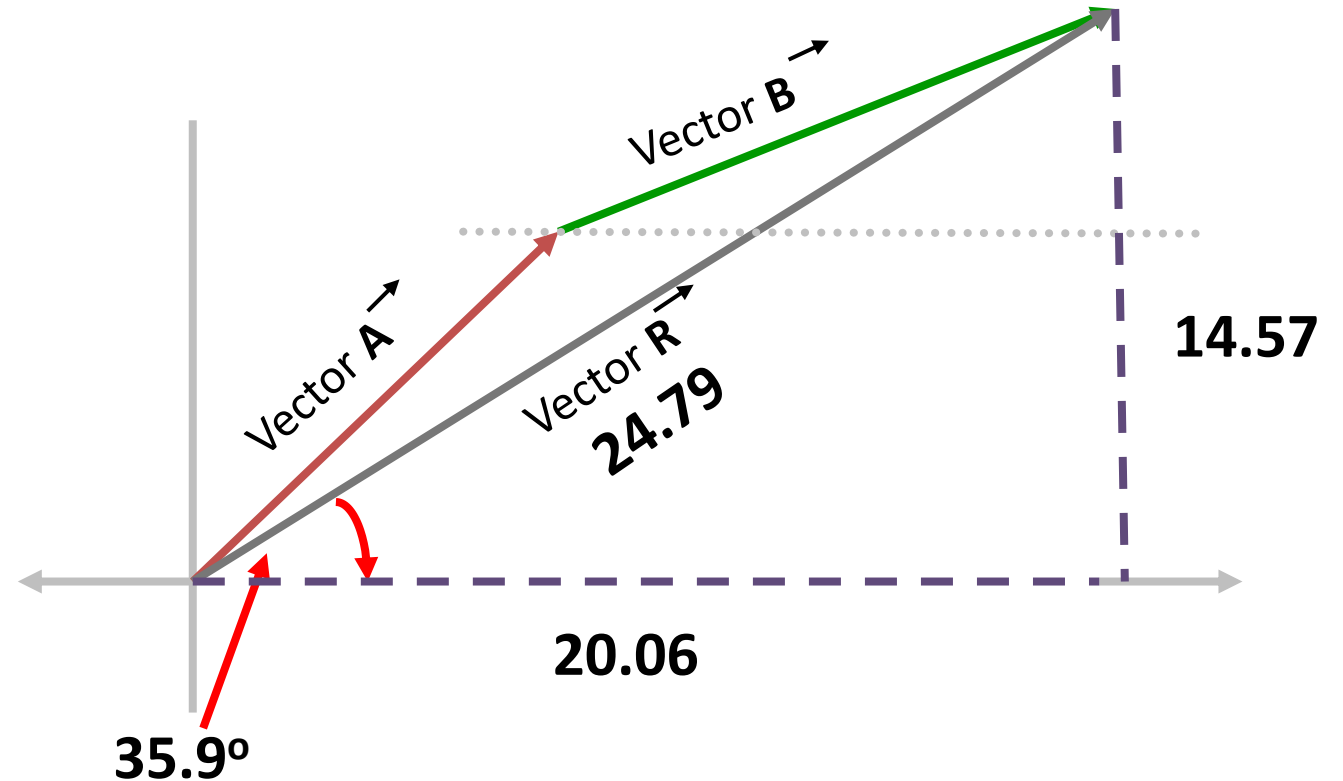
$$\vec{R} = 24.79$$

$$\theta = \sin^{-1}\left(\frac{R_y}{R}\right)$$

$$\theta = \sin^{-1}\left(\frac{14.57}{24.79}\right)$$

$$\theta = \sin^{-1}(.587)$$

$$\theta = 35.9^\circ$$



# Vectors

- Polar notation expresses a vector in terms of both a magnitude and a direction, such as:

*where:*

$$\mathbf{M} \angle ^{\circ}$$

M is the magnitude of the vector  
 $^{\circ}$  is the direction in degrees

***Example:***

Vector with a magnitude of 10 and a direction of -40 degrees  $10 \angle -40^{\circ}$

# Vectors

- Multiplication in polar notation:

Multiply the magnitudes/add the angles

$$(50 \angle 25^\circ)(25 \angle 30^\circ) = 1250 \angle 55^\circ$$

- Division in polar notation:

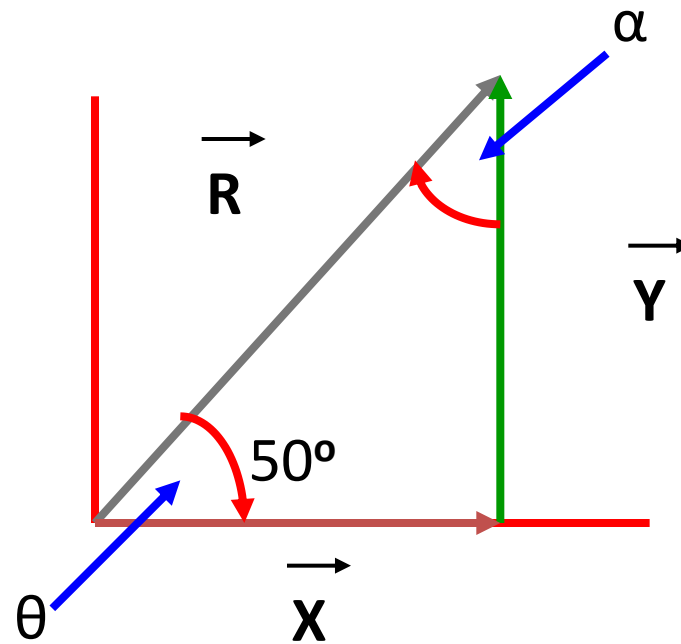
Divide the magnitudes/subtract the angles

$$(50 \angle 25^\circ) = 2 \angle -5^\circ$$

$$\frac{\quad}{(25 \angle 30^\circ)}$$

## Question 9

A displacement vector  $\vec{R}$  has a magnitude of  $R = 175$  m and points at an angle of  $50^\circ$  relative to the x-axis. Find the x and y components of this vector?



## Question 9

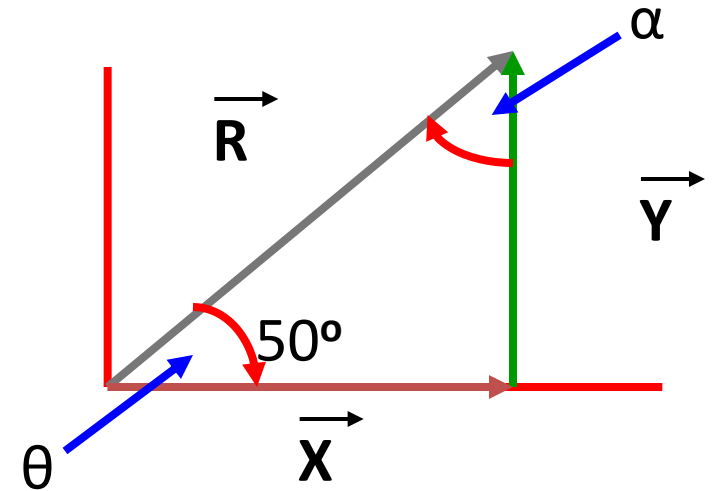
$$\sin \theta = \frac{O}{H} = \frac{Y}{R}$$

$$\sin 50^\circ = \frac{Y}{175m}$$

$$Y = (175m) \sin 50^\circ = 134.06m$$

$$X = \sqrt{H^2 - O^2} = \sqrt{R^2 - Y^2}$$

$$X = \sqrt{175m^2 - 134.06m^2} = \sqrt{12652.92m^2} = 112.5m$$



# Summary

- Discussed the different components of Right Triangles
- Reviewed the basics of Trigonometry
- Computed different Per-Unit Quantities
- Characterized the two components of Vectors

# Questions?

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